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The proofs of Hobson, Wilczynski, Hall and Frink, Paterson (*Elementary Trigonometry*), and Killing and Hovestadt were reduced to a common notation. This article called forth two notes, one from Professor E. R. Hedrick, *School Science and Mathematics*, vol. 16, pp. 347-348, containing a proof of his own which is used by his permission in Kenyon and Ingold, *Plane and Spherical Trigonometry*; and one from Professor C. N. Mills (*loc. cit.*, p. 607), who gives a proof found in "an old text-book of trigonometry."

Professor Lovitt begins by constructing segments for $a + b$ and $a - b$ and then discovering angles equal to $\frac{1}{2}(A + B)$ and $\frac{1}{2}(A - B)$. We can also begin by constructing these angles and then discovering the segments. In this connection I gave in the article cited a proof of my own that I have not seen anywhere in print.

Take $a > b$; let XCY be the bisector of the exterior angle at C , so that $\angle ACY = \frac{1}{2}(A + B)$. Draw $AG \parallel XY$; $AF \perp XY$; $BGD \perp XY$. Then $\angle BAG = \frac{1}{2}(A - B)$.

$$\tan \frac{1}{2}(A - B) = \frac{BG}{GA} = \frac{BD - GD}{DC + CF} = \frac{(a - b) \sin \frac{1}{2}(A + B)}{(a + b) \cos \frac{1}{2}(A + B)} = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B).$$

By adding a few lines to Professor Lovitt's figure I have been able to reduce all the proofs at hand to one figure. From this figure so many other possible proofs appear and the temptation to add still other lines is so great that it seems the prudent thing to rest content with what we have.

III. EXPOSITORY PAPERS FOR THE ASSOCIATION.¹

By E. J. WILCZYNSKI, University of Chicago.

The Mathematical Association of America has reached the conclusion that it can assist very effectively in enlarging the mental horizon of its members by presenting, from time to time, properly conceived papers of an expository character. But the question immediately arises: what is meant by an expository paper, and what are the most desirable characteristics of such a paper? It is my purpose to answer this question very briefly and in a preliminary fashion. Later, when as we hope, a large number of successful expository papers will be available for analysis and comparison, it may become possible to answer this question far more fully.

1. *Choice of Subject.* We all observe, from time to time, in connection with our work of teaching and research, that certain subjects are either omitted entirely from our textbooks, or are treated in inadequate fashion. If it is a subject of general interest and importance which has been thus slighted, it clearly offers a desirable field for an expository paper. Or else we may be interested in some advanced work, and the idea may come to us to explain this work to a non-technical audience. We should follow such impulses, especially when they are

¹ An address delivered before the Mathematical Association of America, and Section L of the American Association for the Advancement of Science at the University of Chicago, December 28, 1920.

strong and spontaneous. But no one should attempt any work of this kind unless at least *he himself* feels strongly that his exposition of the subject fills a real need and constitutes, in some essential way, an improvement on other treatments which are already available.

2. *Prerequisites Required of the Reader.*—The form of the paper will, of course, be determined very largely by the question: for what class of readers are we writing? We may assume that the reader has the usual knowledge of elementary algebra, geometry, and trigonometry; we shall assume further that he has had a first course in analytic geometry and calculus. We must not, however, presume too much on these prerequisites but use our judgment in every case as to whether a certain matter of detail may be passed over without discussion, or whether it requires further explanation. There are plenty of propositions in elementary geometry which even a well trained mathematician may not be able to remember and reproduce on the spur of the moment. In such cases, a reference to a generally accessible book should be given. From analytic geometry and the calculus, we should assume only those propositions which may properly be regarded as being in the possession of all.

3. *Two Kinds of Expository Papers.*—Of course, there exists no absolute line of demarcation between expository papers and other papers. In fact, every mathematical paper is, or should be, expository. But in a research article the emphasis is placed on results and methods, rather than upon any attempt to explain the subject to a large audience. There are at least two kinds of expository papers, namely, those which are purely *descriptive* or *popular*, and those which are also *demonstrative*. That popular papers on mathematical subjects are both possible and desirable is a fact which has not been recognized sufficiently, much to the disadvantage of our science in popular estimation. To reach the maximum of their usefulness, popular papers should contain very full and exact references to those books in which actual proofs may be found for the propositions under discussion. The expository papers presented to the Association will ordinarily be both descriptive and demonstrative. Naturally they will tend to become descriptive, rather than demonstrative, as the subjects with which they deal become more advanced. For it may be quite impossible to discuss a subject demonstratively, within the prerequisites mentioned under section 2, while a descriptive treatment of it would be perfectly feasible.

4. *Definition.*—Every mathematical discipline has certain fundamental concepts which are, in most cases, expressible in the form of definitions. It is vital that these definitions be formulated with great distinctness and clearness. If the reader can only be made to understand these fundamental concepts, the rest is easy.

5. *Rigor and Clearness.*—All proofs given should be mathematically sound and clear. They cannot be clear unless they are sound. It is a fallacy to think that a dishonest proof can help a student. The principal source of obscurity in mathematics, next to logical unsoundness, is excessive brevity. Space is valuable; but excess of brevity, especially in an expository paper, does not save space but

wastes it since it defeats completely the purpose for which the paper was written and, therefore, makes it useless.

6. *Desirability of a Heuristic Treatment.*—The human mind does not care about isolated facts, but likes to gain a *point of view* which will enable it to include many facts at once. Therefore it is essential for the success of an expository paper that the general connection between the problems discussed be made apparent. For this purpose a free, informal style is desirable, and much to be preferred to the formal and dogmatic style of Euclid.

7. *Restriction of Scope.*—If an expository paper is primarily demonstrative, it will necessarily be concerned with a limited field and, in accordance with section 2, this field will have to be of a rather elementary character. Most of such papers, however, will be primarily descriptive. In such descriptive papers no proofs should be attempted except when such proofs can actually be furnished without using other prerequisites than those mentioned in section 2, and when this does not use up too much of the available space, which should properly be reserved for a full and clear exposition of the more important features of the theory. The object of these papers is to *arouse interest* and to convey *some* information, but *not* to carry the reader to the very confines of knowledge. The attempt to accomplish this latter purpose would defeat the former since it would necessarily lead to excessive brevity, thus offending against the principle laid down in section 5.

8. *Character of References Given.*—A reference may be accurate and satisfactory from the point of view of a scholar, and may, nevertheless, prove to be perfectly useless from the point of view of the reader, whom we are attempting to benefit, because the source referred to may be far beyond his comprehension. Therefore, the references should be classified, and special emphasis placed upon those books and articles which are not beyond the reader's scope. This should be supplemented by a short graduated list of standard books, study of which may have the effect of equipping the reader to approach the higher literature of the subject.

RECENT PUBLICATIONS.

REVIEWS.

Gli Scienziati Italiani dall'Inizio del Medio Evo ai Nostri Giorni. Diretto da ALDO MIELI. Volume I, parte 1. Roma, A. Nardecchia, 1921. Royal 8vo. 10 + 234 pages. Price 45 lire.

This is the first part of an important and a sumptuous repertory of which it is expected that one volume will be published each year for a series of years. It is a bio-bibliographical work of Italian scientists from the middle ages to our own day. The term scientists is to be interpreted as including: philosophers, mathematicians, astronomers, physicists, chemists, mineralogists, geologists, biologists, psychologists, geographers, etc. The sketches are all signed and in many of them